Branch-and-Cut and Branch-and-Cut-and-Price Algorithms for the Constrained-Routing and Spectrum Assignment Problem *

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1 Introduction

The Constrained-Routing and Spectrum Assignment (CRSA) Problem can be stated as follows. Let $G = (V, E)$ be an undirected, loopless, and connected graph, which is specified by a set of nodes V, a multiset of links E, and a set of contiguous frequency slots $\{1, \ldots, \bar{s}\}$ with $\bar{s} \in \mathbb{Z}_+$. Each link $e = ij \in E$ is associated with a length $\ell_e \in \mathbb{R}_+$ (in kms), a cost $c_e \in \mathbb{R}_+$. Let K be a multiset of demands such that each demand $k \in K$ is specified by an origin node $o_k \in V$, a destination node $d_k \in V \setminus \{o_k\}$, a slot-width $w_k \in \mathbb{Z}_+$, and a transmission-reach $\bar{\ell}_k \in \mathbb{R}_+$ (in kms). The CRSA consists of determining for each $k \in K$, a (o_k,d_k)-path p_k (subset of edges) in G such that $\sum_{e \in p_k} l_e \leq \bar{l}_k$, and a subset of contiguous frequency slots $S_k \subset \{1, \ldots, \bar{s}\}$ of width equal to w_k such that $S_k \cap S_{k'} = \emptyset$ for each pair of demands $k, k' \in K$ with $p_k \cap p_{k'} \neq \emptyset$ so that the total length of the paths used for routing the demands is minimized. The CRSA is NP-hard as it generalizes the so-called RSA problem which is known to be NP-hard (see [1]). In this work, we introduce two ILP formulations, and provide several classes of valid inequalities for the associated polyhedron. We further discuss their separation problems. Using the polyhedral results and the separation procedures, we devise Branch-and-Cut (B&C) and Branch-and-Cut-Price (B&C&P) algorithms to solve the problem. We also present some experimental results.

2 ILP Formulations

We first introduce an *edge-node* formulation with a polynomial number of variables and an exponential number of constraints which are separable in polynomial time using network flows algorithms. For this, we consider for each $k \in K$ and $e \in E$, a binary variable x^k_e which equals 1 if demand k goes through the edge e and 0 if not, and for each $k \in K$ and $s \in \{1, \ldots, \bar{s}\}$, a binary variable z^k_s which equals 1 if slot s is the last slot allocated for the routing of demand k and 0 if not. The subset of contiguous slots $\{s - w_k + 1, \ldots, s\}$ should be assigned to demand k whenever $z^k_s = 1$. For a node subset X, let $\delta(X)$ denote the set of edges having one endnode in X and the other one in $\bar{X} = V \setminus X$. For a demand $k \in K$, let E^k_0 denote the set of all forbidden edges of demand k such that for each edge $e \in E^k_0$, the length of each (o_k,d_k)-path in G going through edge e exceeds $\bar{l}_k$, and let E^k_1 be the set of essential edges of demand k such that all

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feasible (o_k,d_k)-paths in G, they go through each edge $e \in E_1^k$. The CRSA is equivalent to

$$\min \sum_{k\in K}\sum_{e\in E} l_e x_e^k, \tag{1}$$

$$\sum_{e\in\delta(X)} x_e^k \geq 1, \forall k \in K, \forall X \subset V \text{ s.t. } |X \cap \{o_k, d_k\}| = 1, \tag{2}$$

$$\sum_{e\in E} l_e x_e^k \leq \bar{\ell}_k, \forall k \in K, \tag{3}$$

$$x_e^k = 0, \forall k \in K, \forall e \in E_0^k, \tag{4}$$

$$x_e^k = 1, \forall k \in K, \forall e \in E_1^k, \tag{5}$$

$$\sum_{s=1}^{w_k-1} z_s^k = 0, \forall k \in K, \tag{6}$$

$$\sum_{s=w_k}^{\bar{s}} z_s^k = 1, \forall k \in K, \tag{7}$$

$$x_e^k + x_e^{k'} + z_s^k + \sum_{s'=s-w_k+1}^{min(s+w_{k'}-1,\bar{s})} z_{s'}^{k'} \leq 3, \forall e \in E, \forall k \in K, \forall k' \in K, \forall s \in \{w_k, ..., \bar{s}\}, \tag{8}$$

$$0 \leq x_e^k \leq 1, \forall k \in K, \forall e \in E, \tag{9}$$

$$z_s^k \geq 0, \forall k \in K, \forall s \in \{1, \ldots, \bar{s}\}. \tag{10}$$

Furthermore, we introduce an ILP formulation, called *edge-path* formulation, based on a reformulation of the *edge-node* formulation. For this, we consider for each $k \in K$ and $p \in P^k$ and $s \in \{1, \ldots, \bar{s}\}$, a binary variable $y_{p,s}^k$ which equals 1 if slot s is the last slot allocated along the path p for the routing of demand k and 0 if not, where P^k denote a set of all feasible (o_k,d_k)-paths in G for each $k \in K$. This formulation has an exponential number of variables. A column generation algorithm is then used to solve its linear relaxation.

3 Valid Inequalities and B&C and B&C&P Algorithms

We further identify several classes of valid inequalities to obtain tighter LP bounds. Some of these inequalities are obtained by using conflict graphs related to the problem : clique inequalities, odd-hole and lifted odd-hole inequalities. Wa also use the Chvatal-Gomory procedure to generate larger classes of inequalities. We then devise their separation procedures for these inequalities, and use them to devise B&C and B&C&P algorithms to solve the problem.
We provide a detailed comparative study between our B&C and B&C&P algorithms by using two types of instances : random and realistic ones. They composed of two types of graphs : real graphs, in particular Nsfnet ($|V| = 14$, $|E| = 21$), German ($|V| = 17$, $|E| = 25$) and Spain ($|V| = 30$, $|E| = 56$), and realistics ones from SND-LIB, in particular Europe ($|V| = 28$, $|E| = 41$), France ($|V| = 25$, $|E| = 45$) and German50 ($|V| = 50$, $|E| = 88$). For the demands, the number of demands for each graph varies in $\{10, 20, 30, 40, 50, 60\}$ such that for each triplet $(G, K, \bar{s})$ with a $\bar{s}$ up to 140 slots, we tested 4 instances. The results show that our B&C&P algorithm is able to provide optimal solutions for several instances, which is not the case for the B&C algorithm within the CPU time limit (5 hours). Furthermore, we have studied the influence of our valid inequalities. The results show that some of them, in particular clique and cover inequalities are efficient. However, some instances are still difficult to solve with both B&C and B&C&P algorithms. Our next step is to investigate different branching strategies.

Références

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