LinA : an opensource Julia-based package for piecewise linear approximation of continuous univariate functions with (dis-)continuous piecewise linear functions

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1 Description

We present efficient algorithms able to over-estimate/under-estimate/approximate any arbitrary univariate nonlinear derivable function by a non necessarily continuous (nnc) piecewise linear function that minimizes the number of linear segments with a guaranteed tolerance. The algorithms are based on the piecewise linear bounding method recently proposed by [3] and methodological contributions from [1] that will be presented, such as a generalization of the approach to a larger class of tolerance types than the absolute and relative tolerances from [3].

The package proposed is named LinA and is compatible with JULIA 1.0.1 and onward. It is available at the following address : `http://homepages.laas.fr/sungueve/LinA.html`. It accepts as input the expression of a function, the type of error requested (relative or absolute) and optionally the type of over-/under-/approximation expected (by default approximation) and the algorithm desired (by default it uses the heuristic). It is also possible to define custom error types. Functors are used to make the syntax as natural as possible as shown in FIG 1 that presents a simple example that will be explained during the talk.

```
julia> using LinA

julia> f = Linearize(:(x^2.0+1),-3.0,3.0,Absolute(0.1))
7-element Array{LinA.LinearPiece,1}:
 -5.105572809000085 x -5.416718427000255 from -3.0 to -2.1055728090000847
 -3.316718427000254 x -1.6501552810007603 from -2.1055728090000847 to -1.2111456180001692
 -1.5278640450004222 x + 0.5164078649987369 from -1.2111456180001692 to -0.31671842700025316
 0.26099033699940966 x + 1.0829710109982338 from -0.31671842700025316 to 0.577708763999663
 2.049844718999242 x + 0.04953415699772967 from 0.577708763999663 to 1.4721359549995794
 3.838699100999076 x -2.5839026970027774 from 1.4721359549995794 to 2.366563145999497
 5.627553482998908 x -6.817339551003286 from 2.366563145999497 to 3.0

julia> f[1]
-5.105572809000085 x -5.416718427000255 from -3.0 to -2.1055728090000847

julia> f[1](-2.5)
7.347213595499959

julia> f(2)
5.093495504995374

julia> f[1].xMax
-2.1055728090000847
```

FIG. 1 – Simple example of the LinA syntax

2 Some computational results

Computational results on nonlinear functions approximation benchmark show the efficiency of the package, as illustrated for example on TAB 1 (where '-' means 'results not provided').

input function	δ	continuous approximation			nnc approximation			
		[4]	[5]	[2]	[3]		LinA	
					Exact	Heuristic	Exact	Heuristic
f^1	0.1	Frac. sec.	0.1 s	26 s	-	-	0.006 s	1.3 ms
	0.05	Few sec.	0.1 s	t.o.^{1200s}	-	-	0.003 s	1.6 ms
	0.005	Sec	128.2 s	-	172 s	16 s	0.002 s	1.6 ms
f^2	0.1	Few days	1.9 s	-	77 s	8 s	0.23 s	13 ms
	0.05	t.o.$^{1800s/it}$	12 s	-	64 s	12 s	0.18 s	12 ms
	0.01	t.o.$^{1800s/it}$	13873.4 s	-	114 s	23 s	0.20 s	11 ms
	0.005	t.o.$^{1800s/it}$	42068.4 s	-	784 s	27 s	0.19 s	13 ms

$f^1 : x \to \ln(x)$ with $x \in [1; 32]$; $f^2 : x \to 1.03e^{-100(x-1.2)^2} + e^{-100(x-2)^2}$ with $x \in [-2.5; 2.5]$

TAB. 1 – Computing times for the piecewise linear approximation of nonlinear functions with absolute tolerance δ, i.e. the approximation function l verifies $|f^i(x) - l(x)| \leq \delta, \forall x \in$ definition domain of f^i (also note that $t.o.^k$ refers to a time out reported in the publication for a time limit of k seconds)

LinA can be used to solve certain classes of MINLP problems with linear constraints and a nonlinear objective function separable in a sum of univariate nonlinear terms. TAB 2 shows some results obtained on the congested multicommodity network design problem (cMCND [6]) with multiple nonlinear congestion functions to approximate per instance, one per node of the graph. These preliminary results suggest that results competitive with the state-of-the-art can already be obtained with minimal effort and no consideration on the problem structure.

cMCND instance	Literature ([6])	LinA+MILP
c36_0.8_0.8	21,528 s	14,59 s
c49_0.8_0.6	172,255 s	118,17 s
c50_0.8_0.6	2609,568 s	2575,08 s

TAB. 2 – Results on instances of the multicommodity network design with congestion

Références

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