Minimizing the Maximum Lateness for Scheduling with Release Times and Job Rejection

Imed Kacem[1], Hans Kellerer[2]

[1] LCOMS EA 7306, Université de Lorraine, Metz, France

[2] ISOR, University of Graz, Austria

imed.kacem@univ-lorraine.fr, hans.kellerer@uni-graz.at

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In classical scheduling problems, we are given a set of jobs, all of them have to be processed, and the objective is to minimize a function of the completion times of the jobs. But in many real life production systems, a manufacturer has the possibility to reject some of the jobs and to schedule only the accepted jobs. Therefore, scheduling problems with rejection have been extensively studied in the literature in the last years. As an application example we mention make-to-order production systems with limited production capacity and tight delivery requirements. Another example is the possibility of outsourcing of some jobs to external subcontractors while the jobs can be processed in-house. The reason for rejecting jobs is in many cases that due dates requirements have to be obeyed, i.e. jobs are not allowed to be delivered too late or their lateness must not be larger than a certain bound. On the other side, the total rejection costs shall not exceed the budget limits. Therefore, we investigate in this paper scheduling problems with rejection and the maximum lateness as objective function. More precisely, we have to schedule a set $J = \{1, 2, \dots, n\}$ of n jobs on a single machine or on m parallel machines. Each job $j \in J$ has a processing time p_j, a release time (or head) r_j, a delivery time (or tail) q_j and a rejection cost e_j. Each machine can only perform one job at a given time. Preemption is not allowed. We also define C_j as the completion time of job j, s_j as the starting time of job j, i.e., $C_j = s_j + p_j$ and $P = \sum_{j=1}^{n} p_j$ as the total processing time. For a subset of jobs $J' \subset J$ the value $p(J') = \sum_{j \in J'} p_j$ defines the total processing time and $e(J') = \sum_{j \in J'} e_j$ the total rejection cost, respectively. For a given schedule σ let $A(\sigma)$ and $R(\sigma)$ denote the set of accepted jobs and the set of rejected jobs, respectively. If it is clear from the context, we will write shortly A and R, instead of $A(\sigma)$ and $R(\sigma)$. Let $L_j = C_j + q_j$ denote the lateness of job j. We consider the following two objectives : the first objective is the maximum lateness of the accepted jobs, denoted by $L_{\max} = \max_{j \in A} L_j$. The second objective is the total cost of the rejected jobs, denoted by $RC = \sum_{j \in R} e_j$.

Three types of problems are of interest :

— Problem P1 : minimize the maximum lateness such that the rejection costs are not larger than the upper bound C. This problem is denoted as $1|r_j, \text{rejection}, RC \leq C|L_{\max}$ for a single machine and as $P|r_j, \text{rejection}, RC \leq C|L_{\max}$ for parallel machines, respectively.

— Problem P2 : minimize the rejection costs such that maximum lateness is not larger than the upper bound C. This problem is denoted as $1|r_j, \text{rejection}, L_{\max} \leq C|RC$ for a single machine and as $P|r_j, \text{rejection}, L_{\max} \leq C|RC$ for parallel machines, respectively.

— Problem P3 : find the Pareto-optimal solutions for $L_{\max}$ and RC. This problem is denoted as $1|r_j, \text{rejection}|RC, L_{\max}$ for a single-machine case and as $P|r_j, \text{rejection}|RC, L_{\max}$ for parallel machines, respectively.

Unless stated otherwise, the terms P1, P2 ad P3 shall refer to the single machine problem. Note that solving problem P3 also solves problems P1 and P2 as a byproduct, and of course any non-negative linear combination of the two objective functions.

All three presented problems are generalizations of the well-known problem of minimizing the maximum lateness with release dates on a single machine $1|r_j|L_{max}$ which has been widely studied in the literature. According to Lenstra et al [3] problem $1|r_j|L_{max}$ is NP-hard in the strong sense. Therefore, we are interested in the design of efficient approximation algorithms for our problems.

The main result of this paper is a PTAS for problem P1, which we extend for different variations of problem P1. First, we extend the PTAS for P1 to parallel, identical machines. Then, problem P2 is examined. Afterwards, we give a PTAS for the Pareto scheduling problem P3 and its version with parallel machines. We finish with an FPTAS for the problems $Rm|\text{rejection}, RC \leq C|L_{\max}$ and $Rm|\text{rejection}|L_{\max}, RC$. .

The different results we proved are summarized in the following Table.

Problem	**Results of this paper**
$1\|r_j, \text{rejection}, RC \leq C\|L_{\max}$	PTAS
$1\|r_j, \text{rejection}, L_{\max} \leq C\|RC$	not approximable within ρ factor ($\rho > 1$)
$1\|r_j, pmtn, \text{rejection}, L_{\max} \leq C\|RC$	FPTAS (deduced from [5])
$P\|r_j, \text{rejection}, RC \leq C\|L_{\max}$	PTAS
$P\|r_j, \text{rejection}\|L_{\max}, RC$	PTAS for approximating the Pareto frontier
$Rm\|\text{rejection}, RC \leq C\|L_{\max}$	DP and FPTAS (strongly polynomial)
$Rm\|\text{rejection}\|L_{\max}, RC$	DP and FPTAS (strongly polynomial)
$Rm\|r_j, \text{rejection}, RC \leq C\|C_{\max}$	DP and FPTAS (strongly polynomial)
$Rm\|r_j, \text{rejection}\|C_{\max}, RC$	DP and FPTAS (strongly polynomial)

TAB. 1 – Summary of results

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