The makespan service level in the stochastic flexible-job shop scheduling problem

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1 Introduction

The Flexible Job-shop Scheduling Problem (FJSP) is a generalization of the classical Job-shop Scheduling Problem (JSP), where a set of jobs have to be processed on a collection of machines, and each job requires a number of consecutive operations (routing) before being completed. This problem can be found in many industrial environments, being often subject to various uncertainties induced by machines (e.g., breakdowns), job releases, and/or processing times (e.g., complex multi-chamber machines, tool wear) [3].

Given the aforementioned industrial realities, the current paper focuses on the Stochastic Flexible-Job Shop Scheduling Problem (SJSP) under random processing times. A particular attention is paid to the robustness of the proposed solutions expressed via the concept of makespan service level.

2 Stochastic Flexible Job-shop Problem

The FJSP includes a set of operations partitioned into a set of jobs. The operations in a job have to be executed in a specific order (routing) on a set of available machines. A machine can only perform one operation at a given point in time and cannot interrupt its execution. An operation can be executed by any machine in a given subset. The processing times are machine-dependent. An operation can only be executed once. Finding a solution to this problem is to determine both an assignment of operations to machines, and the sequence the operations follow, while respecting the routing of every job. In this paper, the makespan C_{max} is considered as a required specification, which corresponds to the maximal completion time of all operations. The reader is referred to the literature review of Mönch et al. [4] for a survey of solution approaches of a family of scheduling problems, including the FJSP.

In the FJSP, the processing times are supposed to be known, and set-up times between operations to be negligible or included in the processing times. In the studied SFJSP, processing times are considered unknown and characterized by independent random variables following known finite probability distributions.

3 Notion of makespan service level

By considering the processing times as random variables, the makespan of a solution becomes a random variable, which is commonly aggregated in the form of a scalar value by using the mathematical expectation. To avoid over-conservative solutions for short-term runs, we propose to maximize the probability that the makespan is lower than or equal to a given threshold M. This probability, denoted by $\alpha(S)$, is known as the *service level of the makespan* associated to solution S:

$$\alpha(S, M) = \mathbb{P}(C_{max}(S) \leq M)$$

Very few papers have been dedicated to the notion of service level in scheduling problems. For a single machine scheduling environment, Daniels and Carrillo [5] define a β-robust schedule to represent the likeliness of the total flow time across all jobs to be no worse than a given target level. The notion of β-robustness has been extended by Beck and Wilson [1] to deal with the probabilistic JSP, who also introduce the notion of α-makespan. If the α-makespan of a solution S is lower than or equal to D, then there is at least a $(1 - \alpha)$ probability that the makespan of S is lower than or equal to D.

4 Problem statement and solution approach

The problem under study in this paper aims at maximizing the service level for a specified makespan M. The proposed solution method is based on a competitive state-of-the-art tabu search approach [2], including a Monte Carlo sampling procedure to represent and deal with uncertainties. Monte Carlo sampling methods are known to be computationally expensive, which can quickly became prohibitive when embedded in an optimization framework. For efficiency purposes, several strategies dedicated to accelerating the proposed solution method are proposed.

5 Conclusions and perspectives

Based on extended benchmark instances of the FJSP, extensive computational experiments have been conducted to evaluate the performance of the proposed approach through several prisms: **(i)** The impact of key hyper-parameters of the approach and instances characteristics, **(ii)** The efficiency of the proposed acceleration strategies, and **(iii)** The soundness of the service level with respect to several classical indicators measuring the importance of considering uncertainty.

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