Pollution Routing Problem : A Piecewise-Linear Approximation approach

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Mots-clés : *Pollution Routing Problem, Piecewise-Linear Approximation.*

1 Introduction

In the last decades, several variants of the classical Vehicle Routing Problem (VRP) have been proposed in the literature in order to model more and more realistic operational settings. Among these variants, the well-known Pollution Routing Problem (PRP) aims at finding routes for a homogeneous fleet of vehicles to meet the demands of all customers in a way that a) all the vehicles depart from and return to the depot node, b) each vehicle carries a load less than or equal to its capacity, and c) the service at each customer starts within the respective time window. The objective of the problem is to minimize the overall total cost composed of emissions, operational, and drivers' costs. In particular, consumptions are calculated according to the so-called comprehensive emission model. The PRP is of great importance in the City Logistics context where all the managed operations should pollute as less as possible, however its optimal resolution in practice is much more difficult to achieve with respect to a basic VRP.

2 Contribution

Despite the intrinsic non-linearity of the problem, all the approaches available in the literature (see, e.g., [1] and [2]) are based on a discretization of the vehicles' speed, which allows to model the problem through a Mixed-Integer Linear Programming (MILP) formulation. In this work, instead, we propose a reformulation that exploits a recent JULIA package LinA ([4]) able to provide a non-necessarily continuous piecewise linear approximation of the nonlinear PRP objective function for a desired error tolerance. The method generates, given a desired error tolerance, upper and lower bounding functions minimizing the number of required pieces ([3]). Then, the derived MILP reformulations can be used to efficiently solve the true PRP, thus eliminating the need of resorting to any arbitrary speed discretization and range limitation. Preliminary results show the promising behavior of the approach against benchmark instances.

Références

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