A Clustering-based Simulated Annealing Algorithm for Solving the Uncapacitated Single Allocation Hub Location Problem

El Mehdi Ibnoulouafi, Mustapha Oudani and Mounir Ghogho
International University of Rabat, College of Engineering & Architecture, TICLab, Morocco
{elmehdi.ibnoulouafi,mustapha.oudani,mounir.ghogho}@uir.ac.ma

Mots-clés : *Hub, Location, Clustering, Simulated Annealing, Transportation, Optimisation*

1 Introduction

Hub Location Problems (HLPs) have been extensively studied since their first introduction in [1]. Several real problems from communications networks, air transportation and postal deliveries have led to the introduction of different Hub Location Problem variants to better model each considered Hub-and-Spoke Network. In its classical version, the HLP seeks to open some nodes as hubs while allocating the remaining nodes (spokes) to hubs.
In this work, we introduce a new clustering-based simulated annealing algorithm with a novel integer solution representation in order to solve the Uncapacitated Single Allocation Hub Location Problem (USAHLP). We obtained optimal solutions for all considered small sized instances in small CPU times using the well-known AP (Australian Post) Dataset. Furthermore, our solutions have small gaps to the best known solutions obtained in [4] in competitive computational times.

2 Mathematical Formulation

The authors in [2] proposed one of the most well-known Mixed Integer Programming (MIP) formulations for the Classical Hub Location Problem which seeks to minimize the sum of total hub installation costs, as well as the total cost of transporting all flows between each origin-destination pairs. Let N denote the set of all nodes, F_k the hub fixed installation costs, d_{ij} the unit distance cost, O_i and D_i the flow originated or destined to a node respectively, χ, δ and α the collection, distribution and inter-hub discount factors respectively. Let also s_{ik} denote a binary variable which takes value 1 if and only if a spoke node i is allocated to a hub node k, and Y^i_{kl} denote a variable representing the amount of flow originated from node i circulating from hub k to hub m. We can formulate the aforementioned objective of the Uncapacitated Single Allocation Hub Location Problem as follows :

$$\text{Min} \sum_{i\in N}\sum_{k\in N} d_{ik}s_{ik}(\chi O_i + \delta D_i) + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha d_{kl} Y^i_{kl} + \sum_{k\in N} F_k s_{kk} \tag{1}$$

This objective function is subjected to constraints that guarantee to assign each spoke node to a single hub as well as flow conservation. In order to avoid non-null flows between a pair of hub nodes in case a spoke is not serviced by any hub, we consider the missing cuts that were added by [5].

3 Solution Approach

The classical HLP was proven to be NP-Hard, thus only small sized instances may be solved to optimality using exact methods. In order to solve large sized instances, we implemented a

Simulated Annealing Heuristic which considers the K-Medoids Clustering Algorithm to generate initial starting solutions.
The authors in [3] have previously shown that solving each of the sub-problems involved in the classical hub location problem separately may not be able to achieve optimal solutions for the original problem. So, both the hub location decision and the spoke allocation decision must be tackled jointly. Hence, our implemented heuristic will attempt to craft an initial starting solution by identifying, in each cluster, a hub node for which the allocation cost of all other remaining nodes is minimum in terms of distance. The inter-hub distance will also be considered if a hub node was already located from a cluster in a previous step.
TAB 3 gives a description of the varied swap operations used in the implemented Simulated Annealing algorithm. We note that moves leading to worst cost solutions will be accepted only if the usual acceptation probability usually considered in the implementation for the Simulated Annealing Meta Heuristic is met.

Swap Operation	**Description**
Create Hub	Randomly locate a hub node, then allocate one spoke node to it.
Remove Hub	Randomly remove one hub node, allocate it to another hub node, then allocate its spokes to the remaining hubs.
Change Hub	Select one hub node, then replace it by a spoke node while reallocating all its spoke nodes between the established hubs.
Insert Spoke	Select one spoke node, then allocate it to a different hub node.
Swap Spokes	Select two spokes in two different hubs, then swap their allocation.
Swap Hubs	Select two hub nodes, then swap all their allocated spokes respectively.

TAB. 1 – Swap Operations Description

In an attempt to decrease the complexity of the random swap moves as well as the CPU time required to solve the instances considered for the USAHLP, we have proposed a novel solution representation. The latter consists of replacing the decision variable z_{ik} by a number of data structures, and initializing the inter-hub flow variable Y^i_{kl} only when the solution cost is to be calculated. Also, The computation for both the aforementioned flow decision variables and the solution cost will benefit from the nature of the considered data structures composing the solution, and from the interdependence between them to decrease the amount of operations that would be necessary if the decision variables s_{ik} were to be considered.

Références

[1] O'kelly, M. E. (1986). The location of interacting hub facilities. Transportation science, 20(2) :92-106.

[2] A. T. Ernst and M. Krishnamoorthy, "Solution algorithms for the capacitated single allocation hub location problem,"Annals of operations Research,vol. 86, pp. 141–159, 1999.

[3] Salhi, S. and Rand, G. K. (1989). The effect of ignoring routes when locating depots. European journal of operational research, 39(2) :150-156.

[4] Benaini, A., Berrajaa, A., Boukachour, J., and Oudani, M. (2016). Parallel genetic algorithm for the uncapacited single allocation hub location problem on gpu. In 2016 IEEE/ACS 13th International Conference of Computer Systems and Applications (AICCSA), pages 1–8. IEEE

[5] I. Correia, S. Nickel, and F. Saldanha-da-Gama (2010). "The capacitated single-allocation hub location problem revisited : A note on a classical formulation,"European Journal of Operational Research, vol. 207, no. 1, pp. 92–96.