A practical assessment of MDP and Stochastic Programming approach on the Stochastic Uncapacitated Lot-Sizing problem

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1 Introduction

The single-item deterministic uncapacitated lot-sizing problem (ULS) is a production planning problem first introduced by [5]. It considers a single type of item and aims at determining the quantity to be produced in each time period in order to meet demand over a finite discrete-time planning horizon. Producing a positive amount in a period incurs a fixed cost, called setup cost, together with a production cost per unit produced and an inventory holding cost per unit held in stock between two consecutive periods. The objective is to build a production plan such that the customers' demand is met in each time period and the total costs, i.e. the sum of setup, production, and inventory holding costs over the whole planning horizon, are minimized. In this work, we focus on a stochastic extension of this problem, denoted SULS, in which the problem parameters are subject to uncertainty, i.e., uncertain demand and costs. We consider a multi-stage decision process corresponding to the case where the value of the uncertain parameters unfolds stage-wise following a discrete-time stochastic process and the production decisions can be made progressively as new information on the demand and cost realizations are collected. A review on stochastic lot-sizing problems can be found in [4].

We assume that the underlying stochastic process is stage-wise independent and secondly, that it has a finite probability space. Hence, the uncertain parameters can be represented either by a discrete scenario tree or by a discrete-time Markov Decision Process (MDP). We address this stochastic problem under these two different approaches. Hence, the first resolution approach we consider is based on a representation of the stochastic process by a discrete-time MDP while the second one is based on the stochastic programming modelling. There is a great interest to compare these two approaches since they mainly come from two different research fields. We thus aim at assessing both approaches in terms of quality of solutions and computational scope provided by each method. We discuss their ability to solve the problem as well as their computational limitations. We assess the quality of the optimal policy by comparing the expected costs provided by each method. This is done, under a reasonable computational setting, by carrying out Monte Carlo simulations in a rolling horizon framework.

The MDP approach has been used for many years to model inventory problems, see a review in [2], while capacited and uncapacited lot-sizing problem have been studied in [1] and references therein. Here, our SULS problem is formally described by a discrete time finite horizon MDP which is solved using the library called *marmoteMDP*.

On the other hand, when an approximation of the stochastic process is carried out by a discrete scenario tree, this leads to a formulation of a mixed-integer linear program solved by standard-alone mathematical solvers (*CPLEX* here). However, it is known that this simple scenario tree formulation (as well as MDP models) suffers by the curse of dimensionality making its use computationally impractical for solving instances defined on large-size scenario trees. Thus, an alternative is to use a dynamic dual decomposition approach. This approach decomposes the original formulation into a series of smaller sub-problems linked together by dynamic programming equations. It is based on an extension of the classical Stochastic Dual Dynamic programming algorithm that has been previously used in stochastic lot-sizing problems (see e.g. [6], [3]).

Results Table 1 reports preliminary results of both approaches over very large instances of the SULS. Columns D and N establish the maximum demand and time period horizon for each instance. For the MDP model, Column "Size" reports the size of the MDP matrix needed to create the model. For the stochastic programming approach (SP), Column "# Scen" reports the number of scenarios used to represent the stochastic process. For both approaches, Columns "Time Model" and " Time Solution" report the time (in seconds) to build and to solve the model, respectively. The whole methods had been implemented in C++.

Results suggest that the MDP approach outperforms the stochastic programming approach over the tested instances. The expected costs we obtain are similar for the two models, which indicates the quality of the approximation of the SDDiP.

TAB. 1 – Performance of MDP and SDDiP approaches over large instances of the SULS problem

Instance		MDP			SP (SDDiP)		
D	N	Size	Time Model	Time Solution	# Scen	Time Model	Time Solution
5	4	1200	0.34	0.06	10^{5}	0.03	7.60
	6	1776	0.76	0.22	10^{8}	0.03	433.42
	8	2352	1.33	0.57	10^{11}	0.04	1,520.21
	10	2928	2.09	1.26	10^{15}	0.06	2,958.63
	20	5808	8.28	13.14	10^{31}	0.14	3,844.48
10	4	3960	4.78	0.54	10^{5}	0.08	112.37
	6	5896	10.55	2.14	10^{9}	0.09	2,938.39
	8	7832	18.72	5.80	10^{13}	0.16	3,700.31
	10	9768	29.26	12.42	10^{17}	0.38	3,731.42
	20	19448	115.01	138.45	10^{36}	0.77	3,866.01
20	4	14280	92.76	6.48	10^{6}	0.00	935.89
	6	21336	202.90	26.28	10^{11}	0.01	3,294.92
	8	28392	358.29	69.67	10^{15}	0.01	4,148.31
	10	35448	559.77	149.73	10^{20}	0.01	4,264.63
	20	70728	2,271.32	1,687.15	10^{42}	0.03	4,602.51
Total Result		13268.48	186.38	111.59	10^{40}	0.08	2,378.96

Références

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