Approximation techniques for multi-objective bi-level combinatorial optimization problems

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1 Introduction

Multi-objective Bi-Level Optimization Problems (MBLOPs) are defined as mathematical programs with two nested optimization tasks. In these problems, the evaluation of a single upper level solution requires the evaluation of the lower level Pareto front (PF). This fact increases exponentially the number of the handled decision variables, thereby making the solving process exorbitantly time consuming. To address this problem, researches have been investigated to reduce the computational expense of evolutionary bi-level optimization algorithms using approximation-based techniques such as: numerical optimization within an adaptive exhaustive search method (Eichfelder, 2010), ϵ-constraint and Karush-Kuhn-Tucker based reduction (Shi & Xia, 2001), set-valued mapping (Sinha, Malo, & Deb, 2017), etc. These approaches require exhaustive computational time with several parameters to obtain new PFs. Furthermore, strict mathematical assumptions such as the linearity and the convexity of the problem are needed in these approaches. In this context, Evolutionary Algorithms (EAs) are less sensitive to the problem mathematical properties. However, in the case of real-world applications the optimization process necessitates the execution of a significant number Function Evaluations (FEs). Another fact that could complicate the resolution of MBLOPs is the complex interaction between both optimization levels with discrete variables. For all these reasons, we aim to reduce the induced complexity using approximation techniques in the evaluation process. The main purpose is to incorporate ideas from multi-objective optimization (e.g. Ideal Point (IP)) with the evolutionary process to make EAs efficient in solving combinatorial MBLOPs. We present a recently proposed framework for solving MBLOPs (Abbassi et al., 2020) with the following three types of interaction considering: (1) the whole lower PF in the upper evaluation as in the classical scheme, (2) the maximum compromise solution in the lower PF, or (3) a subset of the whole lower PF.

2 Approximation-based reduction Methods

As mentioned previously, the purpose of this work is to discuss approximation techniques that reduce the processing computational complexity for solving combinatorial MBLOPs. We mention that we identify a Real Surrogate Point (RSP) which presents the maximum similarity rate compared to the IP (cf. Equation 1). The IP can be identified as follows:

$$IP = \min_{x \in PF} f_m(x) \ \forall \ m = 1...M, \text{ x is Pareto optimal} \tag{1}$$

Accordingly, the proposed variants can be summarized as follows:
(1) Return the Complete Lower Evaluation (CLE) PF as in the classical nested EAs.

(2) Return the RSP as Lower level Single Solution Approximation (LSSA): this procedure allows a huge saving in the number of FEs. Besides instead of returning $(x_{ui}, (xl_{j.1},\ xl_{j.2},\ xl_{j.3}))...$ $(x_{ui}, (xl_{n.1},\ xl_{n.2},\ xl_{n.3}))$ $i\ \in\ I,\ j \in\ n$ a single surrogate solution (x_{ui}, xl^s) is returned.
(3) Return a Sub-set Lower Approximation (SLA) using the previous procedure iteratively.

3 Experimental study

The proposed variants are applied to a multi-objective bi-level production-distribution planning problem using the pr suite of instances (Cordeau, Gendreau, & Laporte, 1997). Table 1 shows that the LSSA algorithm outperforms the SLA and CLE variants in 6 over 10 used instances in terms of averaged fitness value. This fact can be explained by the focus on the upper level decision variables in the evaluation step. The use of the reference point allows huge savings in the number of FEs by evaluating one representative solution instead of the whole lower PF. Indeed, LSSA obtains usually the less number of FEs (cf. Figure 1) for pr04. We note that we choose the pr04 as a comparison based benchmark because it represents a medium size instance with 100 retailers. To conclude, experimental results revealed the efficiency of LSSA and SLA when compared to traditional bi-level multi-objective EAs. Encouraged by these promising results other approximation-based techniques hybridized with the chemical reaction optimization algorithm (Abbassi, Chaabani, & Said, 2021) are currently in development.

TAB. 1: Upper level average distance.

Instance	LSSA	SLA	CLE
pr01	**3310**	3250	3415
pr02	**5910**	5940	5980
pr03	11647	11778	**11644**
pr04	**12940**	12976	12996
pr05	11918	11870	**11864**
pr06	18024	17998	**17991**
pr07	**5160**	5317	5203
pr08	**9917**	10233	10193
pr09	13632.	**13524**	13740
pr10	**20714**	20940	20862

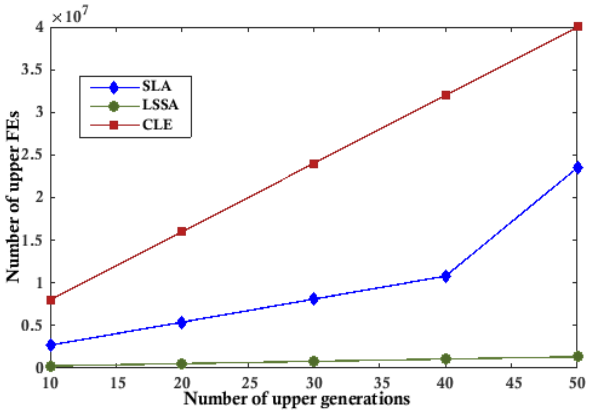

FIG. 1: Numbers of FEs obtained on the pr04.

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