The Benders by batch algorithm: design and stabilization of an enhanced algorithm to solve multicut Benders reformulation of two-stage stochastic programs

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1 Context and problem solved

We study the resolution of large-scale two-stage stochastic linear programs with Benders decomposition [Benders, 1962]. The resolution of such problems arises in many applications such as network design, facility location or vehicle routing problems. RTE (Réseau de Transport d'Électricité) solves such problems in order to perform forecast report on the French electricity network to horizons of 5 to 15 years. Benders decomposition is a state of the art method to solve them [Rahmaniani et al., 2017]. Since this algorithm can be desperately slow to converge, a large amount of research has been performed, but there is still no generic method to accelerate its convergence.

We focus on problems with continuous first-stage and second-stage variables. We assume that the probability distribution is given by a set of scenarios and focus on problems with a large number of scenarios. We study the idea of solving only a few subproblems at each iteration of the algorithm, as solving all the subproblems at each iteration can be time-consuming when the total number of subproblems is large. This idea has been already studied [Dantzig and Infanger, 1991; Higle and Sen, 1991; Oliveira et al., 2011] but the proposed methods either needed restrictive assumptions on the uncertainty or lost the optimality of the algorithm. We also study the use of *in-out* stabilization, which has already shown good numerical results in different situations, see [Pessoa et al., 2013; Fischetti et al., 2016]. Many papers studying the resolution of large LPs with Benders decomposition use stabilizations based on bundle methods [Oliveira et al., 2011].

2 Contribution

Our contribution is an exact algorithm to solve the multicut Benders reformulation of a two-stage stochastic linear program, the Benders by batch algorithm, without any restrictive assumption on the structure of the uncertainty. We study the idea of solving only a few subproblems at each iteration of the algorithm. We show that classical stopping conditions and primal stabilization methods cannot be applied directly to our method, and propose new ones, whose validities are shown formally. The numerical efficiency of our method is proven through an extensive numerical study on six classical benchmark instances from the stochastic literature. We compare our algorithm to five methods : an implementation of classical Benders decomposition algorithm (see Algorithm 1), declined in its monocut and multicut variants, a Benders decomposition algorithm with in-out stabilization [Ben-Ameur and Neto, 2007], also declined in its monocut and multicut variants, and the built-in Benders decomposition of CPLEX 12.10. We show acceleration ratios of approximately 8 times faster on average than the best classical method we compare to, and up to 800 times faster than IBM ILOG CPLEX 12.10 built-in Benders decomposition, as shown in [FIG. 1].

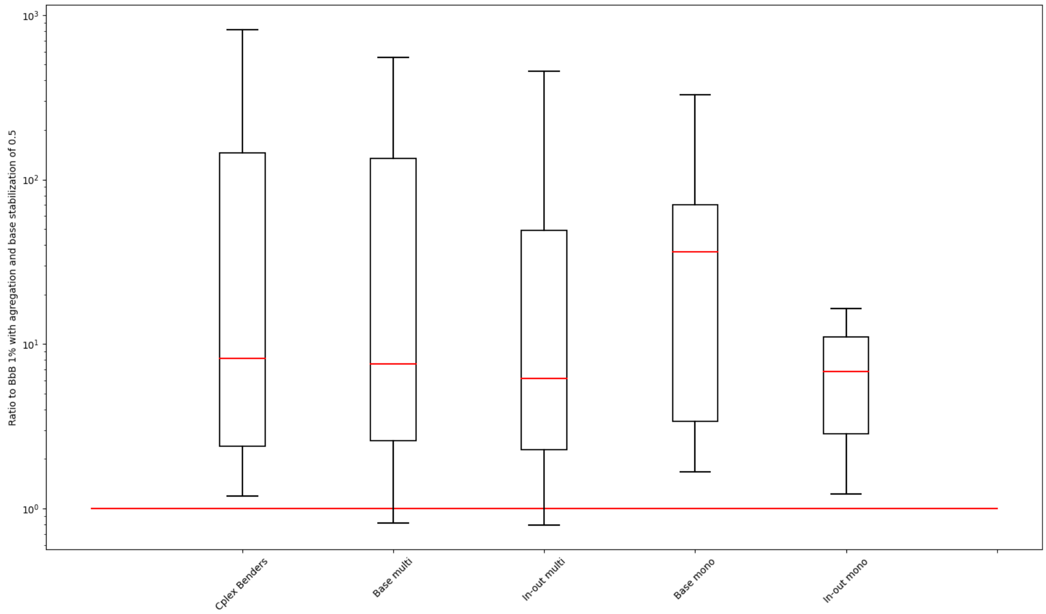

FIG. 1 – Repartition of the ratios between resolution times of our best stabilized method and the five initial methods we compared to, on our 72 hardest instances. The bottom line of each box-plot shows the ratio min, the upper line, the ratio max, the red line shows the median. The lower and upper lines of the boxes show the repartition between the 25% lowest ratio and 75% lowest ratio.

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