An iterative approach for set-union knapsack problem

Isma DAHMANI[1], Meriem FERROUM[1], Mhand HIFI[2]

[1] AMCD-RO, Département de RO, USTHB, BP 32, El Alia,
16111 Bab Ezzouar, Algiers, Algeria
{dahmani.isma, meriem1.mf}@gmail.com

[2] EPROAD, UPJV, 7 rue du Moulin Neuf, 80000 Amiens, France
hifi@u-picardie.fr

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1 Introduction

In this study, a variant of the well-known binary Knapsack Problem, namely KP, is tackled ; that is the Set Union Knapsack Problem (Goldschmidt et al. [1]), namely SUKP. An instance of SUKP is composed of n elements and m items, where elements related to item i is denoted by P_i and $\cup_{i=1}^{m} P_i = \{1, \ldots, n\}$. Each item i has a profit p_i, each element j has a weight w_j and there is a knapsack constraint with capacity b. For $K \subseteq \{1, \ldots, m\}$, we define $P_K = \cup_{i \in K} P_i$ and the formal description of SUKP may be given as follows :

$$\max \Big\{ \sum_{i \in K} p_i \ \Big| \ \sum_{j \in P_K} w_j \leq b, \ K \subseteq \{1, \ldots, m\} \Big\}.$$

2 A hybrid swarm optimization

The SUKP is tackled with a hybrid swarm optimization-based algorithm, where a basic Particle Swarm Optimization (PSO) is combined with a local search. The proposed approach may be summarized as follows :

A basic swarm intelligence PSO is a population-based heuristic, which ensures approximate solutions for a given problem (Eberhart and Kennedy [6]). In this work, a solution is represented by a vector $\vec{x}$ of positions on the space. Each particle $x_{(i)}$ is characterized by two elements : (i) the fitness value that is related to the objective value and (ii) the velocity that drives the particle in the search space. At each iteration, a particle i is updated by using previously information related to both the best fitness at hand ($p_{Best}^{(i)}$, with velocity $\vec{v}(i)$) and the best objective value of the population (g_{Best} : the global best solution). Each particle shares information with other particles and increments its positions such that

$$v_{(i)}^{t} = \omega \times v_{(i)}^{t-1} + c_1 \times \upsilon \times \left[p_{Best}^{i} - x_{(i)}^{t-1} \right] + c_2 \times \nu \times \left[g_{Best} - x_{(i)}^{t-1} \right] \tag{1}$$

and

$$x_{(i)}^{t} = x_{(i)}^{t-1} + v_{(i)}^{t}, \tag{2}$$

where Eq. (1) updates the velocity of a particle i for the t-th iteration and Eq. (2) updates its position. The parameters c_1 and c_2 represent the cognitive and social factors, respectively where $c_1 + c_2 \leq 4$, where both υ and ν are randomly generated in the interval $[0, 1]$.

Particule's enhancement. The used strategy can be viewed as a two-stage procedure :

— *The first stage :* it corresponds to an intensification search, where Arulselvan's procedure is applied (Arulselvan [5]). Note that a simple improvement can be applied to any Greedy Solution Procedure (GSP). Herein, GSP starts by fixing the first item founded by the formulation $\max_{i \in I} \left\{ \frac{p_i}{\omega_i} \right\}$, where $\omega_i = \sum_{j \in I_i} \left(\frac{w_j}{d_j} \right)$ and d_j denotes the number of occurrences of the j-th element in I_i. During the process, it gives a chance to each item, so that it can belong to at least a feasible solution.

— *The second stage :* a 2-opt operator is employed as a local search applied to a solution in order to improve its quality. For a given solution, the 2-opt operator repeatedly makes some moves as long as it improves the quality of the current solution. Herein, we propose to use an intensified 2-opt operator which mimics a critical item used in the single binary knapsack problem (Al-Douri *et al.* [2]).

3 Preliminary results

A preliminary study was conducted on some benchmark instances extracted from Wei and Hao [4]. The results achieved by the proposed approach was analyzed and compared to those published in more recent papers of the literature.

	Results from [3, 4]		This work	
#Inst	Av.	Best	Av.	Best
sukp 185_200_0,10_0.75	13696,00	13696	13696,00	13696
sukp 185_200_0,15_0.85	11298,00	11298	11298,00	11298
sukp 300_285_0,10_0.75	11568,00	11568	11730,90	**12111**
sukp 300_285_0,15_0.85	11799,27	11802	11802,00	11802
Av	12090,31	12091	12131,72	12226,75
Sukp 900_900_0.10_0.75	9729,51	9745	9738,20	9745
Sukp 900_900_0.15_0.85	8918,96	8990	8975,20	8990
Sukp 1000_1000_0.10_0.75	9431,47	9544	9541,30	**9551**
Sukp 1000_1000_0,15_0.85	8376,20	8474	8473,50	**8538**
Av	9114,03	9188,25	**9182,05**	**9206**

TAB. 1 – Behavior of the iterative method versus two recent methods of the literature

Table 1 reports the results, on some instances, achieved by the proposed approach and those reached by two methods([3] and [4]). Column 1 of the table displays the instance label, columns 2 and 3 report the best bounds and the best average bounds (throughout ten trials for each method) achieved by both methods, and columns 4 and 5 tally those achieved by the proposed method. From Table 1, one can observe that the proposed method is very competitive, since it is able to provide three new bounds and matches the other ones for the instances tested.

Références

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