The 2-machine robust flow shop problem under budgeted uncertainty

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Mots-clés : *optimisation, ordonnancement, flow shop, robuste.*

1 Introduction

We study scheduling problems in which associated equipment undergoes a series of maintenance operations that must have their execution order respected. In particular, we investigate a variation of the problem known as *Permutation Flow Shop*. In flow shop scheduling problems, the operations are performed in a specific sequence (serial flow). Such configuration is commonly used in assembly lines throughout several types of industry, including chemical, electronic, food and metallurgical. Even though flow shop scheduling has been widely investigated considering deterministic processing times, in real-world situations, the decision making process can be affected by uncertainties in operation processing times.

In order to deal with these uncertainties, a normal approach in the literature to tackle this problem is using Stochastic Optimization [3], i.e., assigning probabilities to different scenarios in order to generate a decision which is optimal in the long run. However, this is not an option when the probability distributions for the processing times are unknown. Instead, Robust Optimization (RO) [1] can be applied, with the objective of finding a solution that is good enough considering the realization of all possible scenarios, minimizing the objective function considering the worst-case realization.

Based on Robust Optimization techniques, this work proposes new solution approaches to the flow shop problem where operation processing times are subject to uncertainty. The problem under study is the 2-machine Robust Permutation Flow Shop (2RPFS) problem to minimize the worst-case makespan. The novelty of our work is the application of the budgeted uncertainty set [2], which allows to adjust the level of conservatism of the solution. The objective is to provide exact solutions to this variation of the problem.

2 The Permutation Flow Shop (PFS) Problem

The PFS problem can be stated as follows. Consider a production planning process consisting of a set $J = \{J_1, J_2, \ldots, J_n\}$ of n jobs to be executed in a set $M = \{M_1, M_2, \ldots, M_m\}$ of m machines. In this process, every job J_j is composed by m stages $O_{1,j}, O_{2,j}, \ldots, O_{m,j}$, named operations. Every operation $O_{r,j}$ has a non-negative processing time $t_{r,j}$ forming the matrix $T \in \mathbb{R}^{+}_{M \times J}$. The job operation $O_{r,j}$ must only be executed on machine r. At any time, each machine cannot execute more than one operation. Operation $O_{r,j}$ can be executed only after operation $O_{r-1,j}$ is finished. Preemption is not allowed, that is, once an operation is started, it must be completed without any interruption. The sequence in which the jobs are to be processed is the same for each machine. Such order is defined by a permutation $\pi : \{1, \ldots, n\} \longrightarrow J$, with $\pi(j)$ indicating the $j-th$ job to be executed. The completion time of an operation $O_{r,j}$, denoted by $C_{r,j}$, can be defined by the recurrence :

$$C_{r,\pi(j)} = \begin{cases} t_{r,\pi(j)} & \text{if } r = 1 \text{ and } j = 1 \\ C_{r,\pi(j-1)} + t_{r,\pi(j)} & \text{if } r = 1 \text{ and } j > 1 \\ C_{r-1,\pi(j)} + t_{r,\pi(j)} & \text{if } r > 1 \text{ and } j = 1 \\ max(C_{r,\pi(j-1)},\ C_{r-1,\pi(j)}) + t_{r,\pi(j)} & \text{if } r > 1 \text{ and } j > 1 \end{cases}$$

The completion time of a job J_j is $C_{m,j}$. The *makespan* of a permutation is the maximum completion time of a job (C_{max}). The objective of the PFS problem is to find a sequence of jobs (permutation π) which minimizes the *makespan*.

3 Solution approach

Concerning the uncertain nature of the problem, scenarios are used to represent the set of possible realizations of processing time values t. In order to apply RO, there are two usual ways of describing the set of scenarios. In the discrete case, an explicit scenario list is given, while in the interval case each processing time t can take any value between a lower ($\underline{t}$) and an upper bound ($\bar{t}$). Our research is based on the interval case.

Regarding the interval scenario case, to our knowledge, the only existing work that deals with the degree of solution conservatism is the one by Ying [4], which proposes two metaheuristics for the 2RPFS problem, in which processing times can take any value according to the budgeted uncertainty set introduced by Bertsimas and Sim [2]. Our research fills a gap in the literature with an exact solution method for this problem. We propose a column-and-constraint generation algorithm, based on the framework proposed by Zeng & Zhao [5], including two main elements :

— Robust counterpart models based on the best-performing PFS problem formulations ;

— Worst-case calculation procedure ;

By developing this exact algorithm, we expect to provide more flexible solutions, thus enabling the decision-maker to evaluate the trade-off between robustness and solution cost.

Acknowledgement

Research conducted during a visiting period at Avignon Université by means of a Doctoral Exchange Program sponsored by CAPES, Ministry of Education, Brazil (Process No. : 88881.187708/2018-01).

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