Optimal Location of Safety Landing Sites in urban air mobility

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The development of new urban air mobility services is one of the major challenges for transport companies. Using airspace will reduce congestion, traffic jams and waste of time moving from a source site to a destination in urban context. Indeed flying taxi (Vertical Take Off and Landing Vehicules: VTOLs) would exploit vertical space to reduce congestion on the ground and allow passengers to click on an app to order a shared flight. Aerial ride sharing will enable reliable transportation through a network of small electric VTOLs (eVTOLs) that can take off and land vertically. VTOL planes must be safer than cars because the loss of human life in the event of an accident is much greater. In emergence, VTOLs search a spare landing site i.e. safety landing site (SLS) to land. If the trajectory of a VTOL is not covered by installed SLS, routing passengers on that trajectory would be dangerous.

In this work, we study the problem of optimizing the placement of SLS while minimizing the operational cost of routing VTOLs, the costs of installed SLSs and satisfying the security requirements.

Therefore, the first safety constraint stipulates that every part of the routing trajectory is covered by at least one SLS. The second safety constraint stipulates that a bounded number of VTOLs can fly in the same area in case of congestion. According to the airline regulation, the number of VTOL in the same area cannot exceed this bound, and we call this bound as the capacity. Owing to the budget constraint on SLS installation, the operator aim to place a bounded number of SLSs that meets the safety requirements.

We propose multi-commodity flow formulations to this problem. We assume that VTOLs cruise at a fixed height due to airline regulations, so the transportation network is represented as a two-dimensional geometric graph. Edges of the network are available trajectories for the airlines, and their capacities define the maximum number of VTOLs in the edge, which approximates the capacity in the neighborhood of the edge. Part of nodes of the network represent the skyports for VTOLs to take off or land. SLSs are not nodes, they are installed between nodes such that they cover sets of edges. Feasible routing for a VTOL satisfies the safety constraint and the capacity constraints. Each VTOL takes a path and has a capacity on the number of passengers. The operator defines the daily demands, and optimization experts need to design the installation of SLSs, and assign passengers to VTOLS such that the routing cost is minimized.

Unsplittable multi-commodity flow and k-splittable multi-commodity flow play a basic role in the VTOL traffic model. Multi-commodity flow problem can be represented by a flow on edges formulation or a flow on path formulation. The path formulation is a Dantzig-Wolfe reformulation of the edge formulation. We develop two traffic models derived from $k-$splittable and unsplittable multi-commodity flow problems, and for each model edge and path formulations are proposed.

Path formulations contain exponential numbers of path variables but have less constraints. We solve their LP relaxations by column generation approach. To enforce the integrality, we use the branch-and-price algorithm based on our column generation method. The implementation of the path formulation for $k-$splittable model needs columns and rows generation during the pricing loop. We provide numerical experiments for the performance of the branch-and-cut algorithm, using Cplex, for the two models and the branch-and-price algorithm, with SCIP, for the unsplittable multi-commodity flow model.

We discuss trade-offs between path and edge formulations for solving the SLS network design problem. To conclude, we give future ideas on robust cuts, and combination of the Bender decomposition and the branch-and-price-and-cut approach.

Keywords— Network Optimization, Mixed Integer Linear Programming (MILP), Network Design, Integer Multi-Commodity Flow, Branch and Price, Column Generation.