A POPMUSIC matheuristic for the capacitated vehicle routing problem and its variants

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1 Introduction

The *Capacitated Vehicle Routing Problem* (CVRP), is one of the most widely studied problems in combinatorial optimization and operations research. The CVRP is the prototypical vehicle routing problem. New ideas are often first proposed and tested on CVRP and then generalized to other vehicle routing variants.

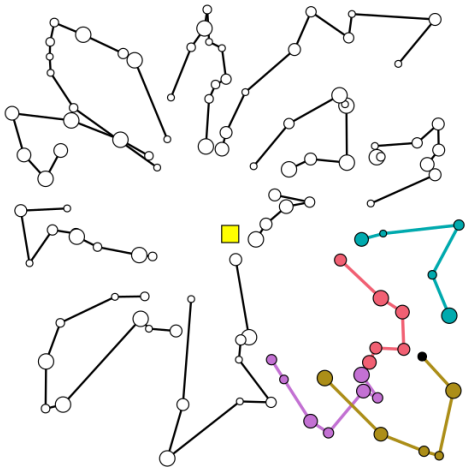
Given that CVRP is NP-hard, most of the algorithms proposed for this problem are heuristics and metaheuristics [3]. On the other hand, the exact methods for CVRP have advanced considerably in recent years [1]. The algorithms that hybridize metaheuristics with mathematical programming approaches [2] are often known as *matheuristics*. One of the types of matheuristics is based on the decomposition of the original problem into smaller subproblems that can be solved (optimally or sub-optimally) through mathematical programming models. This work proposes a simple Partial OPtimization Metaheuristic Under Special Intensification Conditions (POPMUSIC) [5] for the CVRP that uses a modern branch-cut-and-price (BCP) solver [4] to solve subproblems. The algorithm is simple enough to be relatively easily extended to other vehicle routing problems by exploiting the generality of the BCP solver.

2 Algorithm

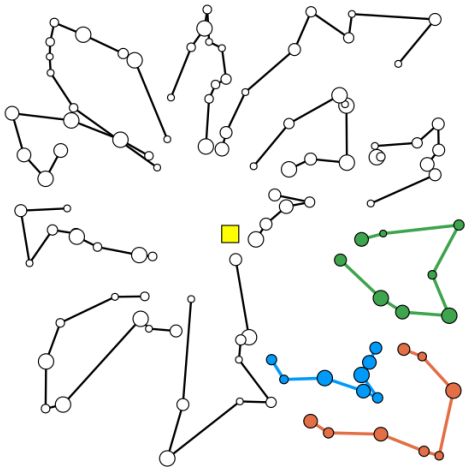
Due to the lack of space, we give only a general description of the algorithm. The general idea is to exploit the following property of the modern exact BCP algorithms for the CVRP. Although the exponential growth of the solution time is still there when the instance size increases, small and medium-size instances can be solved rapidly when a very good upper bound is available. In this case, many instances with up to 150 customers can be solved to optimality in few minutes, and many instances with up to 100 customers can be solved in seconds.

The algorithm is designed to improve a good initial solution even further, possibly in a long run. On each iteration of the algorithm, we form a restricted problem of a size not exceeding the current *target dimension*, and solve it to (near) optimality by the BCP algorithm. We keep track of already solved restricted problems in order not to solve them again. The target dimension is initialized with a small value (50 clients), and then gradually increased during the execution of the algorithm. This is done so that the algorithm solves smaller restricted

problems first (easy even with not so good upper bounds), so the solution of larger restricted problems can benefit from already improved upper bounds. An illustration of an iteration of the algorithm is depicted in Figure 1.



(a) Initial solution and the constructed restricted problem.



(b) Improved solution after finding a better sub-solution.

FIG. 1 – An iteration of our POPMUSIC algorithm for the CVRP.

3 Results

The computational results show that our approach outperforms one of the best published metaheuristics for the CVRP [6] in medium and long runs (from 15 minutes to 32 hours). The results are especially good for instances with relatively short routes. Moreover, several best known solutions were improved for literature instances with up to 20,000 customers, see `vrp.atd-lab.inf.puc-rio.br/index.php/en/updates`. This shows a very good scalability of the approach. At the conference, we are also going to present results for variants of the CVRP : the vehicle routing problem with backhauls and the heterogeneous vehicle routing problem.

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