Introducing the Resilient Ring Star Problem

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Several network design, telecommunication, transportation and facility location problems among many others, involve designing networks in a tributary or backbone architecture. Different design of tributary and backbone networks exist, see for instance, Klincewicz [4]. We consider the Ring Star network design, where we are given a complete mixed graph with both arcs from and to every node, as well as edges between any pair of different nodes and a fixed node called depot. The Ring Star Problem (RSP) consists in selecting a subset of nodes including the depot, named hubs, and link them with a cycle to form the ring. Then each non-hub node is connected to exactly one hub in the cycle, that is the star topology part (see Figure 1 for an example).

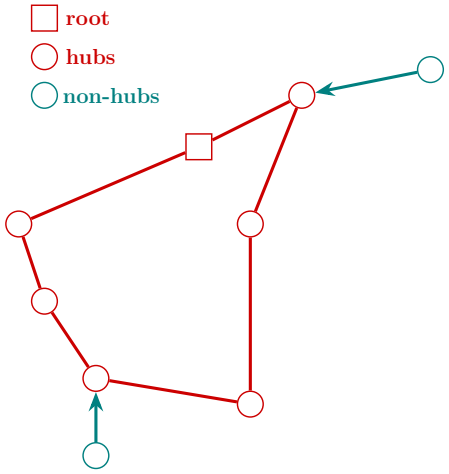

FIG. 1: A solution for a small RSP instance of 9 nodes.

The aim of RSP is to minimize the sum of three costs corresponding to (i) selecting the subsets of hubs, (ii) linking the ring, and (iii) connecting the star. RSP has been widely studied in the literature, Labbé *et al.* [5] proposed a Mixed Integer Programming model, strengthened with valid inequalities studied with a polyhedral analysis and solved with a Branch-and-Cut algorithm. Another exact resolution that takes advantage of the fact that the depot must be in the ring can be found in Kedad-Sidhoum and Nguyen [3]. Calvete *et al.* [2] have presented an evolutionary-based heuristic for solving RSP while Zang *et al.* [6] studied a recent ant colony system algorithm.

In this paper, we study the Resilient Ring Star Problem (RRSP) in the case where at most one hub can fail at any time. The resilient ring-star network is designed so that when a hub fails, two corrective operations occur. A ring correction consists of restoring the ring by adding a backup edge that joins the two neighbors of the failing hub and a star repair operation aims to connect the non-hub nodes that were originally connected to the failing hub to another hub with backup arcs. Figure 2a illustrates a solution of RRSP on the same instance introduced previously where regular edges and arcs are shown in solid lines while backup arcs and edges are dashed. Figure 2b presents the final ring-star structure resulting from the failure of the top right hub.

The objective of RRSP is to minimize the sum of four costs: (a) The cost of selecting hubs that will be part of the ring; (b) The cost of connecting all non-hubs nodes to the ring with regular arcs; (c) The cost of regular edges to form the ring; and (d) The maximum cost incurred by the corrective operations that occur when a single hub fails.

In a practical setting, the regular edges that define a solution to RSP should be used for a time period whose duration is known (typically one year). Once deployed, backup edges are

used to cope with the failure of a single hub, which incurs a cost that depends on the duration of the failure, so backup costs are expressed in monetary unit per unit of time. Finally, the decision maker has to provide the parameter F, which is the total amount of time during which at most one hub will be down during the time period. Hence the proposed solution to RRSP minimizes the total cost incurred by the deployment of the solution and the repairing operations that can occur during the time period, provided that the total failing time does not exceed F. A similar use of such a parameter can be found in Section 3 in Bertsimas and Sim [1]. This parameter expresses the risk protection that the decision maker wishes to achieve. Low values of F will favor a solution to RSP with low regular cost (RRSP reduces to RSP when $F = 0$), whereas high values for F will tend to minimize the maximum failing cost in the RSP solution.

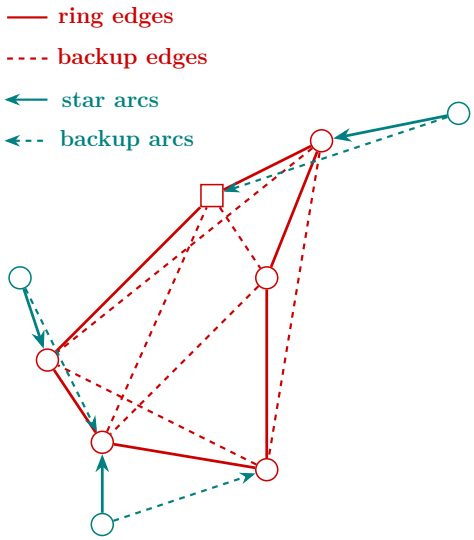

FIG. 2a: A solution for the RRSP on the example of Figure 1.

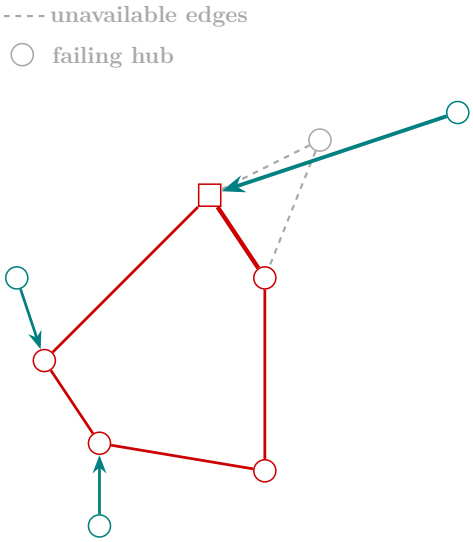

FIG. 2b: The RSP solution that results if the grey hub fails.

To the best of our knowledge, RRSP has not been studied yet in the literature. We have currently formulated and implemented a Mixed Integer Linear Programming model for RRSP.

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