Solving a smart-charging problem with the quantum algorithm QAOA on gate-based quantum computers

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1 Introduction

Quantum Approximation Optimization Algorithm (QAOA) recently emerged as a promising variational approach to approximately solve combinatorial optimization problems with gate-based quantum computers of the NISQ [1] era [1]. It becomes more and more important to qualify its performance in real-world applications. We addressed this challenge with an industrial problem from the high-growth sector of electric vehicles proposed by EDF. The goal is to tailor and tune the quantum metha-heuristic for the studied problem and to qualify the performance of the resulting solution.

The numerical analysis was realised on data of about 2250 loads performed on Belib network.

2 Smart charging problem

We consider a set of recharge jobs with specified priorities that we wish to schedule on parallel stations such that the total weighted completion time is minimized. More precisely, given K identical recharge stations the goal is to find a schedule for N recharge jobs with fixed durations $T = [t_1, \ldots, t_N]$ and priorities $W = [w_1, \ldots, w_N]$ that minimizes the total completion time:

$$C = \sum_{i=1}^{N} w_i C_i \tag{1}$$

The problem is (weakly) NP-complete and admits an (F)PTAS [2] (for fixed K) [3]. Thus, we can compute the exact optimal values and fairly evaluate the solutions produced by QAOA.

In our approach we use the formulation of the initial problem in terms of Max-K-Cut problem on a complete weighted graph $G = (V, E)$ with $|V| = N$ and $E : (u, v) \to \min\{w_i t_j, w_j t_i\}$. The Max-K-Cut problem is NP-complete and if $P \neq NP$ it cannot be approximated with a factor $\alpha > \frac{16}{17}$. If, in addition, the highly believed *Unique Game Conjecture* holds, the bound becomes $\alpha > 0.84\ldots$ [2] which is matched by the Goemans-Williamson algorithm.

2.1 Methodology

In quantum algorithms for combinatorial optimization an instance of the problem is mapped an energy operator defined on a set of possible solutions called Hamiltonian $\mathbf{H}$. The ground state of $\mathbf{H}$ corresponds to the optimal solution. A quantum-classical heuristic QAOA returns a low-energy energy state that approximate the optimum of the objective function.

[1] Noisy Intermediate Scale Quantum devices that are expected to be available in the next 10 years
[2] Fully Polynomial-time Approximation Scheme

QAOA has a polynomial runtime with the constant factor that depends on a tunable depth parameter p. The quantum part consists of p applications of parametrized blocks where the parameter values are optimized by a classical routine. Increasing depth in theory improves the quality of the solution (in fact, as follows from the *adiabatic theorem* in the limit $p \to \infty$ QAOA finds the exact optimum) but it signigicantly complexifies the search of the optimal parameters so in practice the applications are limited to small values of p ($p \sim 10$).

By introducing binary encoding we extend the original QAOA, designed to for our usecase approximately solve combinatorial problems of type *QUBO*[3]:

$$\min f : \{0,1\}^n \to \mathbb{R}_+ \tag{2}$$

We also compare different methods for the searching for QAOA optimal parameters and establish that the Nelder-Mead numerical optimization is the best choice while the widely-used COBYLA method is unable to find the good parameter values. We compile all observations in an experimental protocol dedicated to the QAOA solving our problem.

3 Conclusion and future work

As QAOA is an heuristic, we can't proof a general analytical bound on its approximation ratio. However, the numerical analysis shows that QAOA outperforms the randomized algorithm and that the results improve while the depth is growing. We observe that even at lowest depth $p = 1$ on graphs of all considered sizes QAOA demontstrates the average approximation ratio that is bigger than 0.85 - the ratio achieved by Goemans-Williamson classical algorithm. In future

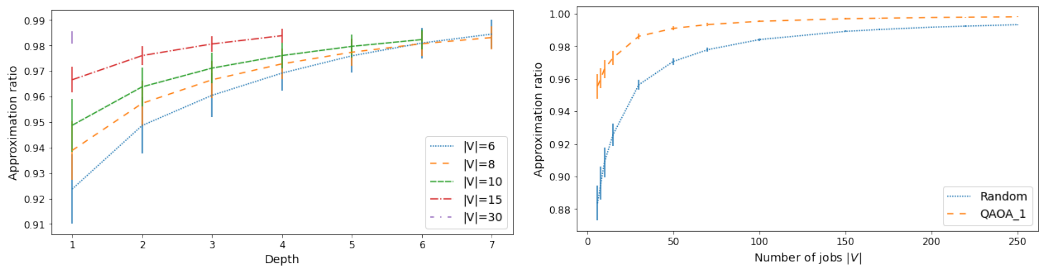

FIG. 1: Left panel: Evolution of the approximation ratio of *QAOA* with depth p for the Max-Cut problem. Right panel: Evolution of the average approximation ratio with the instance size for *QAOA* at depth $p = 1$ (dashed orange line) and for the randomized algorithm on the *initial scheduling problem* (dotted blue line).

work we plan to explore the performance of QAOA on more realistic models for smart charging as well as to study other quantum algorithms such that *Quantum Annealing* or *RQAOA*.

References

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