A Branch-and-Cut algorithm to solve the multi-commodity flow blocker problem

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1 Introduction

We are interested in evaluating the strength of a network, by determining the maximum number of failures that it can face.
A network is represented as a graph $G(V, A)$ where V is the set of nodes and A is the set of arcs. Each arc $a \in A$ has a maximum capacity c_a. This graph is associated with a set of commodities D. Each commodity $d \in D$ is a triplet (s_d, t_d, b_d) where $s_d \in V$ is the source, $t_d \in V$ the destination and $b_d \in \mathbb{Z}^+$ the bandwidth, i.e the flow value passing from s_d to t_d.
A multi-commodity flow problem consists in finding a set of arcs routing flow in order to satisfy all demands, respecting flow conservation constraints and capacity constraints. The solution is denoted as a feasible multi-commodity flow.
The **multi-commodity flow blocker problem** is a bi-level[1] optimization problem where a blocker[2] problem, also called master, applies to a multi-commodity flow problem, also called follower. It consists in finding the minimal number of arcs that have to be destructed from the network such that the multi-commodity flow problem is not feasible.

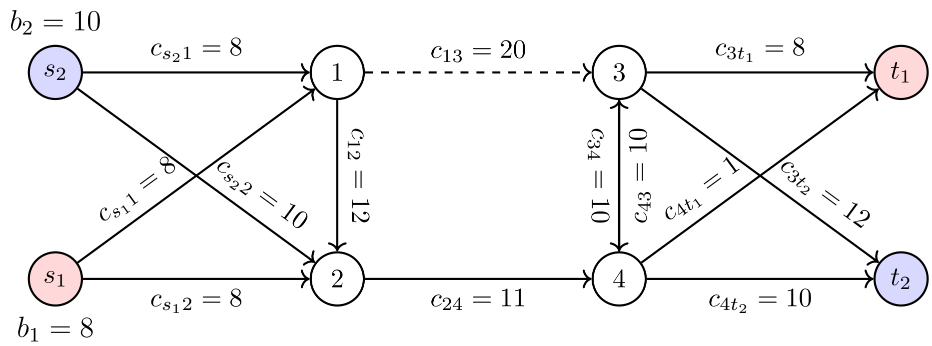

FIG. 1: Example of a multi-commodity flow blocker problem

Figure 1 shows an example of a multi-commodity flow blocker problem applied to a simple graph with 8 nodes, 12 arcs and 2 commodities $\{(s_1, t_1, 8), (s_2, t_2, 10)\}$. Solution of the multi-commodity flow blocker problem is represented with dashed lines. Remark that by deleting arcs $(1, 3)$, the multi-commodity flow problem is not feasible.

In telecommunication networks, a failure is represented by the destruction of an arc. Therefore, the main goal of our work is to find the maximum number of arcs for which the deletion does not impact demands satisfaction. We can notice that adding one unit to this number leads to the minimum number of arcs for which the deletion aims to destruct entirely the routing. This represents the solution of a multi-commodity flow blocker problem.

2 Solution approach

We introduce a new IP model for the multi-commodity flow blocker problem. Our goal is to provide a thin and well-understood formulation.
Let $B \subset A$ be the solution of the multi-commodity flow blocker problem, i.e the set of arcs that will be removed from the network. This solution is valid if it intercepts at least one arc of each feasible multi-commodity flow in the network. To this end, for each arc $a \in A$, a binary variable z_a is defined such that :

$$z_a = \begin{cases} 1 \text{ if } a \in B \\ \quad 0 \text{ else.} \end{cases}, \forall a \in A \tag{1}$$

The multi-commodity flow blocker problem can be modeled as follows :

$$\min \quad \sum_{a \in A} z_a \tag{2a}$$

$$\sum_{a \in A(mcf)} z_a \geq 1 \qquad\qquad mcf \in MCF \tag{2b}$$

$$z_a \in \{0,1\} \qquad\qquad a \in A, \tag{2c}$$

Where MCF is the set of feasible multi-commodity flow in the network and $A(mcf)$ is the set of arcs composing mcf. The objective function (2a) minimizes the number of deleted arcs. Constraints (2b), denoted as *cover constraints* guarantee that all feasible multi-commodity flow in the network will be covered by removing at least one arc from each of them. In other words:

$$|B \cap A(mcf)| \geq 1, \forall mcf \in MCF \tag{3}$$

We note that the number of cover constraints is exponential. Thus, we use a branch-and-cut algorithm to solve this model.

3 Conclusion

The multi-commodity flow blocker problem has many applications, especially in the resilience analysis of a telecommunication network. To solve this problem, we propose an integer model that will be solved with a branch-and-cut algorithm. For our further proceedings, we investigate new inequalities to strengthen the model. We also plan to study variants of the problem. For example, we can study the case of one commodity and make a relation with well-known optimization problems. Finally, we will extend the work for traffic anomalies.

References

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