Upper Dominating Set : Tight Algorithms for Pathwidth and Sub-Exponential Approximation

Louis Dublois[1], Michail Lampis[1], and Vangelis Th. Paschos[1]

Université Paris-Dauphine, PSL University, CNRS, LAMSADE, Paris, France
{louis.dublois,michail.lampis,paschos}@lamsade.dauphine.fr

Mots-clés : *complexité, approximation, pathwidth, upper dominating set*

1 Introduction

In a graph $G = (V, E)$ with $|V| = n$ and $|E| = m$, a set $D \subseteq V$ is called a *dominating set* if all vertices of V are dominated by D, that is for every $u \in V$ either u belongs to D or u is a neighbor of some vertex in D. The well-known Dominating Set problem is studied with a minimization objective : given a graph, we are interested in finding the smallest dominating set. Here, we consider *upper dominating sets*, that is dominating sets that are minimal, where a dominating set D is minimal if no proper subset of it is a dominating set, that is if it does not contain any redundant vertex. We study the problem of finding an upper dominating set of maximum size, and this problem is called Upper Dominating Set.

Upper Dominating Set was first considered in an algorithmic point of view by Cheston et al. [1], where they showed that the problem is NP-hard. In the more extensive paper considering this problem, Bazgan et al. [2] studied approximability, and classical and parameterized complexity of the Upper Dominating Set problem. In the polynomial approximation paradigm, they proved that the problem does not admit an $n^{1-\varepsilon}$-approximation for any $\varepsilon > 0$, unless P = NP, making the problem as hard as Max Independent Set, whereas there exists a greedy $\ln(n)$-approximation for the Min version Min Dominating Set.

Considering the parameterized complexity, they proved that the problem is as hard as the Independent Set problem : Upper Dominating Set is W[1]-hard parameterized by the standard parameter k. Nonetheless, in their reduction, there is an inherent blow-up in the size of the solution k, so they essentially proved that there is no algorithm solving Upper Dominating Set in time $O(n^{o(\sqrt{k})})$. They also gave FPT algorithms parameterized by the pathwidth pw and the treewidth tw of the graph, in time $O^*(7^{pw})$ [1] and $O^*(10^{tw})$, respectively.

2 Results and Methods

The state of the art summarized above motivates two basic questions : first, can we close the gap between the lower and upper bounds of the complexity of the problem parameterized by the pathwidth ; second, since the polynomial approximation is essentially settled, can we design sub-exponential approximation algorithms which can reach any approximation ratio $r < n$? We answer these questions and along the way we give stronger FPT hardness results.

In fact, we prove the following :

— First, we show that, under ETH, there is no algorithm solving Upper Dominating Set in time $O(n^{o(k)})$; and under the same complexity assumption, for any constant ratio r and any $\varepsilon > 0$, there is no algorithm for this problem that outputs an r-approximation in time $O(n^{k^{1-\varepsilon}})$. To obtain these two FPT hardness results, we make a linear fpt reduction from the Independent Set problem to our considered Upper Dominating Set problem,

1. O^* notation suppresses polynomial factors in the input size.

and we rely on the two following hardness result for the former problem : under ETH, Independent Set cannot be solved in time $O(n^{o(k)})$ [3] ; under ETH, for any constant ratio $r > 0$ and any $\varepsilon > 0$, there is no r-approximation algorithm for Independent Set running in time $O(n^{k^{1-\varepsilon}})$ [4].

— Then, we give a dynamic programming algorithm parameterized by the pathwidth pw that solves Upper Dominating Set in time $O^*(6^{pw})$. Surprisingly, this result is obtained by slightly modifying the algorithm of Bazgan et al. [2]. We then prove the following : under SETH, and for any $\varepsilon > 0$, Upper Dominating Set cannot be solved in time $O^*((6-\varepsilon)^{pw})$. Obtaining such lower bounds for pathwidth and treewidth under the SETH was initiated by Lokshtanov et al. [5], and originally made via a reduction from an instance of 3-Sat. Here, we start with an instance of q-CSP-6, and we rely on a more recent result of Lampis [6], stating that under the SETH, for any $\varepsilon > 0$, there exists a q such that q-CSP-6 cannot be solved in time $O^*((6 - \varepsilon)^n)$.

— Finally, we give a simple time-approximation trade-off : for any ratio $r < n$, there exists an algorithm for Upper Dominating Set that outputs an r-approximation in time $n^{O(n/r)}$. We also give a matching lower bound : under the randomized ETH, for any ratio $r > 1$ and any $\varepsilon > 0$, there is no algorithm that outputs an r-approximation running in time $n^{(n/r)^{1-\varepsilon}}$. To obtain this inapproximability result, we rely on the inapproximability result of Chalermsook et al. [7] stating that for any $\varepsilon > 0$ and any sufficiently large $r > 1$, if there exists an r-approximation algorithm for Independent Set running in time $2^{(n/r)^{1-\varepsilon}}$, then the randomized ETH is false.

Références

[1] Cheston G.A., Frickle G., Hedetniemi S.T., Jacobs S.T. : On the computational complexity of upper fractional domination. Discret. Appl. Math. 27(3), 195-207 (1990).

[2] Bazgan C., Brankovic L., Casel K., Fernau H., Jansen K., Klein K., Lampis K., Liedloff K., Monnot J., Paschos V.T. : The many facets of upper domination. Theor. Comput. Sci. 717, 2-25 (2018).

[3] Chen J., Huang K., Kanj I.A., Xia G. : Strong computational lower bounds via parameterized complexity. Comput. Syst. Sci. 72(8), 1346-1367 (2006).

[4] Bonnet E., Escoffier B., Kim E.J., Paschos V.T. : On subexponential and fpt-time inapproximability. Parameterized and Exact Computation - 8th International Symposium, IPEC 2013, pp. 54-65 (2013).

[5] Lokshtanov D., Marx D., Saurabh S. : Known algorithms on graphs of bounded treewidth are probably optimal. Proceedings of the Twenty-Second Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2011, pp. 777-789 (2011).

[6] Lampis M. : Finer tight bounds for coloring on clique-width. 45th International Colloquium on Automata, Languages, and Programming, ICALP, vol. 107, pp. 86 :1-86 :14 (2018).

[7] Chalermsook P., Laekhanukit B., Nanongkai D. : Independent set, induced matching, and pricing : Connections and tight (subexponential time) approximation hardness. 54th Annual IEEE Symposium on Foundations of Computer Science, FOCS 2013, pp. 370-379 (2013).