Non-smooth optimization in complex numbers to solve the ACOPF's dual problem

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1 Introduction

The Alternating-Current Optimal Power Flow (ACOPF) problem models the optimisation of electricity dispatching in power systems. This is an active research topic, which interests both the power systems and operations research communities. The ACOPF has indeed multiple applications for the management of electrical grids and despite the considerable research work on the topic, solving realistic instances to global optimality is still a challenge.

In a steady-state AC network, the state variables of the system, e.g. the voltage at every bus or the current in each line, are sine signals. Hence, they are represented by complex variables and this is why the ACOPF problem is an optimization problem "natively" expressed in complex numbers. More precisely, it can be formulated as a Quadratically-Constrained Quadratic Programming (QCQP) problem with complex decision variables [1]. The dual problem of the ACOPF is a semi-definite programming (SDP) problem, which is in fact the dual SDP problem of the ACOPF's rank relaxation [6]. Both primal and dual SDP relaxations of the ACOPF are expressed in complex numbers with a $n \times n$ Hermitian matrix, but can be written as real SDP problems with a $2n \times 2n$ real symmetric matrix thanks to an isometry argument. State-of-the-art methods use a clique decomposition technique, which exploits the sparsity of electrical networks, to solve the ACOPF's SDP primal-dual relaxation in real variables [7, 8].

The SDP relaxation of the ACOPF problem is known to produce tight lower bounds [4], but it is admittedly not the most tractable type of relaxation [5]. The work presented here is a research effort to improve the efficiency of solving the ACOPF's SDP relaxation. The originality of this work is to focus on the solution of the dual problem by addressing its "natural" formulation in complex numbers. The proposed solution algorithm is a bundle method that exploits the sparsity of the problem in a very different way from the clique decomposition technique.

2 The ACOPF Hermitian spectral dual

Although the decision vector $x \in \mathbb{R}^m$ in the dual ACOPF problem has real components, this dual problem contains a Linear Matrix Inequality (LMI) involving an Hermitian matrix $\mathcal{A}(x) \in \mathbb{C}^{n\times n}$. This explains why this formulation cannot be given as such to a standard SDP solver. Unlike the state-of-the-art methods solving large-scale instances of the ACOPF's SDP relaxation, we proceed without converting the SDP problem in real variables. In this respect, we are in line with the work of Gilbert and Josz [2], who advocate for methods to solve directly the original formulation of a SDP problems in complex variables. We exploit the structure of the affine operator $x \mapsto \mathcal{A}(x)$ involved in the complex LMI and prove that the dual ACOPF problem can be written as a non-smooth concave optimization problem in real variables and with real affine constraints. The objective function of this new formulation that we propose here involves the minimum eigenvalue of the Hermitian matrix $\mathcal{A}(x)$. In this regard, we named this formulation the Hermitian spectral dual of the ACOPF problem.

3 A bundle method to solve the Hermitian spectral dual

The use of bundle methods for eigenvalue optimisation has already been widely explored in cases where the max (or min) eigenvalue function is applied to real symmetric matrices [3]. To the best of our knowledge, this work is the first to investigate the use of a bundle method to solve a Hermitian eigenvalue problem. This method exploits the sparsity of the network through the use of dedicated routines for extreme eigenvalue computation of sparse Hermitian matrices. We present extensive numerical results, based on different choices for the proximal regularization and for the lower approximation model. These results are promising and demonstrate the relevance of solving the Hermitian formulation of the dual ACOPF. It also shows that large-scale instances of the ACOPF's SDP relaxation can be solved without having recourse to the clique decomposition technique. Further research avenues concern the use of more sophisticated bundle methods (like $\mathcal{UV}$-bundle methods) and the extension of this approach to the sum-of-squares (SOS) hierarchy in complex variables.

References

[1] Daniel Bienstock, Mauro Escobar, Claudio Gentile and Leo Liberti. *Mathematical programming formulations for the alternating current optimal power flow problem.* 4OR 18.3 (2020): 249-292.

[2] Jean-Charles Gilbert and Cedric Josz. *Plea for a semidefinite optimization solver in complex numbers.* Technical Report hal-01422932, HAL Archives-Ouvertes, 2017.

[3] Christoph Helmberg and Franz Rendl. *A spectral bundle method for semidefinite programming.* SIAM Journal on Optimization 10.3 (2000): 673-696.

[4] Cédric Josz, Stéphane Fliscounakis, Jean Maeght, and Patrick Panciatici. *AC power flow data in MATPOWER and QCQP format: iTesla, RTE snapshots, and PEGASE.* arXiv preprint arXiv:1603.01533, 2016.

[5] Burak Kocuk, Santanu Dey, and Andy Sun. *Strong SOCP relaxations for the optimal power flow problem.* Operations Research 64.6 (2016): 1177-1196.

[6] Javad Lavaei and Steven Low. *Zero duality gap in optimal power flow problem.* IEEE Transactions on Power Systems 27.1 (2011): 92-107.

[7] Ramtin Madani, Abdulrahman Kalbat, and Javad Lavaei. *ADMM for sparse semidefinite programming with applications to optimal power flow problem.* 2015 54th IEEE Conference on Decision and Control (CDC). IEEE, 2015.

[8] Daniel Molzahn, Jesse Holzer, Bernard Lesieutre, and Christopher DeMarco. *Implementation of a large-scale optimal power flow solver based on semidefinite programming.* IEEE Transactions on Power Systems 28.4 (2013): 3987-3998