Basic Rounding Strategy for k-Clustering Minimum Bi-clique Completion Problem

Mhand Hifi[1], Shohre Sadeghsa[1]

Unité de Recherche EPROAD, Université de Picardie Jules Verne
7 rue du Moulin Neuf - 80000 Amiens, France
{hifi,shohre.sadeghsa}@u-picardie.fr

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1 Introduction

The *k-Clustering Minimum Bi-clique Completion* problem (noted P_{KCMBC}) is initially studied by Faure *et al.* [1]. They defined this problem in a case study in the telecommunication field. It is defined in a bipartite graph. In this study, we adopt two definitions from the terminology of graph theory to describe P_{KCMBC}.
Bipartite graph : A graph $G(S, C, E)$ is bipartite, where S represents the services, C clients and E denotes the edges linking the items from C to S in the noted bipartite graph.
Complete bipartite graph : A graph $G(S, C, E)$ is called a complete bipartite graph if for any vertices $s_1 \in S$ and $c_1 \in C$, $s_1c_1 \in E$. This graph has $|S| + |C|$ vertices and $|S| * |C|$ edges.

P_{KCMBC} is to decompose a complete bipartite graph $G(S, C, E)$ into K clusters so that each sub-cluster will be a complete bipartite graph.

2 Basics of the rounding strategy

We recall the mathematical formulation of P_{KCMBC} from (cf., Faure *et al.* [1]) in (1)-(3) :

$$\omega(x) = \min \sum_{k\in K} \sum_{(i,j)\in \bar{E}} x_{ik} y_{jk} \tag{1}$$

$$\text{s.t.} \sum_{k\in K} x_{ik} = 1, \forall i \in S \tag{2}$$

$$x_{ik} \le y_{jk}, \quad \forall (i,j) \in E, \forall k \in K \tag{3}$$

$$x_{ik}, y_{jk} \in \{0,1\}, \quad \forall i \in S, \forall j \in C, \forall k \in K,$$

In this non-linear integer formulation of P_{KCMBC}, x_{ik} and y_{jk} are defined as binary variables as {1 : if the service i (vertices from S and respectively client j in C) is assigned to cluster k, $k \in K$; 0 otherwise}.

The goal of P_{KCMBC} is to (Equality (1)) minimize the number of edges ($\bar{E}$ indicates the edges that do not belong to the set E) that needs to be added to new clusters to compose a complete bipartite subgraph. Equation (2) implies that each service $i \in S$ should appear in only one cluster whereas a client $j \in C$ can appear in more than one cluster. If there is an edge in set E that connects a vertices $i \in S$ to $j \in C$, equation (3) ensure that a client $j \in C$ is assigned to the same cluster as its corresponding services in S. The mathematical formulation of P_{KCMBC} and its linear model is also proposed in [3]. The authors applied a branch-and-price algorithm along with an accelerating method to converge the solution founded by Cplex. This problem has been studied in several works. An algorithm based on variable neighborhood search, proposed in [7] and rounding strategy with different enhancement methods in [4, 5, 6].

The rounding procedure is defined on the mixed-integer linear formulation of P_{KCMBC}. Since the problem is complex, we first solved the problem with the semi-relaxed formulation defined in 4 to 8.

$$
\begin{aligned}
\omega'(x) = \min \quad & \sum_{k\in K}\sum_{(i,j)\in\bar{E}} z_{ijk} \\
\text{s.t.} \quad & x_{ik} + y_{jk} \leq 1 + z_{ijk}, \quad \forall (i,j)\in\bar{E}, \forall k\in K && (4)\\
& \sum_{k\in K} x_{ik} = 1, \quad \forall i \in S && (5)\\
& x_{ik} \leq y_{jk}, \quad \forall (i,j)\in E, \forall k \in K && (6)\\
& 0 \leq x_{ik} \leq 1,\ 0 \leq y_{jk} \leq 1, \quad \forall i\in S, \forall j \in C, \forall k\in K, && (7)\\
& z_{ijk} \in \{0,1\}, \quad \forall (i,j)\in\bar{E}, \forall k\in K, && (8)
\end{aligned}
$$

The semi-relaxed model could find solutions for P_{KCMBC} if all the variables x_{ik} and y_{jk} were binary. However, in the proposed relaxed model, there is no constraint to enforce them to be binary. Herein a feasible solution for P_{KCMBC} can be reached by rounding variables of a solution from 4 to 8 that have the largest fractional value by respecting the rest of the constraints, i.e., constraints from 4 to 8. This process is repeated until there is no more fractional value. The resulted feasible solution is then enhanced with the hill-climbing strategy using local operators and two scatter strategies embedded in an iterative search for implementing a powerful method. The proposed method is tested on the benchmark instances with $|S| = |C| \in \{15, 18, 20, 50, 80, 100, 120, 150, 200, 300\}$ and $|k|$ in $\{5, 10, 15\}$.

Conclusions and perspectives In this study, we described the basics of the rounding strategy applied to P_{KCMBC}. The proposed strategy is capable to be combined with other algorithms to enhance their efficiency. The proposed method was evaluated and compared to the best methods available in the literature on the benchmark instances. New bounds have been obtained.

Références

[1] Faure, N., Chrétienne, P., Gourdin, E., & Sourd, F. (2007). Biclique completion problems for multicast network design. Discrete Optimization, 4(3-4) :360-377.

[2] Hifi, M. (2014). An iterative rounding search-based algorithm for the disjunctively constrained knapsack problem. Engineering Optimization, 46(8) :1109-1122.

[3] Gualandi, S., Maffioli, F., & Magni, C. (2013). A branch and price approach to k-Clustering minimum biclique completion problem. International transactions in operational research, 20(1), 101-117.

[4] Hifi, M., & Sadeghsa, S. (2020). Adaptation of the rounding search-based algorithm for the k-clustering minimum completion problem, In IEEE, the 7th International Conference on Control, Decision and Information Technologies (CoDIT), Prague, Czech Republic, pp. 1150-1155 (doi : 10.1109/CoDIT49905.2020.9263846).

[5] Hifi, M., & Sadeghsa, S. (2021). An Iterative Randomized Rounding Algorithm for the k-Clustering Minimum Completion Problem with an Application in Telecommunication Field, Accepted in Computing Conference 2021, London, UK.

[6] Hifi, M., & Sadeghsa, S. (2021). A Rounding Strategy-Based Algorithm for the k-Clustering Minimum Biclique Completion Problem, Under revision in Journal of the Operational Research Society.

[7] Al-Iedani, N., Hifi, M., & Saadi, T. (2016, April). Neighborhood search-based heuristic for the k-clustering minimum biclique completion problem. In 2016 International Conference on Control, Decision and Information Technologies (CoDIT) (pp. 639-643). IEEE.