An ϵ-Constraint Strategy-Based Method for the Bi-Objective Quadratic Multiple Knapsack Problem

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1 Introduction

The knapsack problem arises in several applications, like packing, logistics and multimedia. Often simple deterministic and stochastic approximate solution procedures may be used for tackling some types of these problems. However, it has been remarked that the aforementioned approaches may converge towards non-desirable solutions.

In this study, we investigate the use of the two-stage ϵ-constraint strategy-based method for solving the Bi-Objectif-Quadratic Multiple Knapsack Problem (BO-QMKP). Such a problem can be viewed as a combination of two well-known NP-hard combinatorial optimization problems, where two objective functions are considered : (i) quadratic knapsack problem and (ii) multiple knapsack problem.

2 The problem

An instance of BO-QMKP is characterized by a set $M = \{1, \ldots, m\}$ of m knapsacks of fixed capacity each, i.e., $c = (c_1, \ldots, c_m)$, and a set $N = \{1, \ldots, n\}$ of n items. Each item i, $\forall\ i \in N$, is characterized by a profit p_i and a weight w_i such that each pair of items (i, j) belonging to $N \times N$ $(i \neq j)$ has an augmented profit p_{ij} if both items belong to the same knapsack $k,\ k \in M$. The goal of the problem is to assign each item to at most one knapsack such that the total weight of the items in each knapsack $k,\ k \in M$, does not exceed its capacity c_k and both (i) the total profit of all the items included into the knapsacks and (ii) the makespan related to the knapsack with the lowest gain, are maximized. Formally, the BO-QMKP can be stated as follows :

$$z_1(x) = \max \quad \sum_{i\in N}\sum_{k\in M}^{m} p_i x_{ik} + \sum_{i\in N}\sum_{\substack{j\in N\\ i<j}}\sum_{k\in M} p_{ij} x_{ik} x_{jk} \tag{1}$$

$$z_2(x) = \max \min_{k\in m} \sum_{i\in N} p_i x_{ik} + \sum_{i\in N}\sum_{\substack{j\in N\\ i<j}} p_{ij} x_{ik} x_{jk} \tag{2}$$

$$\text{s.t.} \qquad \sum_{i\in N} x_{ik} w_i \leq c_k, \forall k \in M, \tag{3}$$

$$\sum_{k\in M} x_{ik} \leq 1, \forall i \in N, \tag{4}$$

$$x_{ij} \in \{0,1\}, \forall i \in N,\ \forall j \in M, \tag{5}$$

3 The proposed method

The ϵ-constraint can be used as an alternative strategy that can be able to generate a series of non-dominated points. The provided solutions can be considered as the solutions forming the Pareto front. Indeed, the used process may mimic the following steps : let $\underline{z}_1$

(resp. $\underline{z}_2$) be the best objective function provided by the local branching LB of [4]). Then, the following optimization problem can be considered :

$$z_1(x) = \max \sum_{i \in N} \sum_{k \in M}^{m} p_i x_{ik} + \sum_{i \in N} \sum_{\substack{j \in N \\ i<j}} \sum_{k \in M} p_{ij} x_{ik} x_{jk} \tag{6}$$

$$\max \min_{k \in m} \sum_{i \in N} p_i x_{ik} + \sum_{i \in N} \sum_{\substack{j \in N \\ i<j}} p_{ij} x_{ik} x_{jk} \leq \underline{z}_2(x) - \epsilon \tag{7}$$

$$\text{s.t.} \qquad (3),\ (4)\ \text{and}(5). \tag{8}$$

Of course, the above problem can be transformed whenever the second objective function z_2 should be optimized.

Herein, a preliminary computational results is provided to assess the performance of the proposed method, where its provided results are compared to the best known bounds available in the literature. The instances used as benchmarks are taken from Chen *et al.* [4], where a set of 15 large-scale instances are considered with $n = 300$ (items).

TAB. 1 – Behavior of the ϵ-constraint method versus some best available methods

		#Inst			SO	IRTS [2]	EPR [3]		BSSBM [1]		Iϵ-CH	
n	d	m	I	c	$\underline{z}_1$	$\underline{z}_1$	$\underline{z}_1$	$\underline{z}_2$	$\underline{z}_1$	$\underline{z}_2$		
300	25	3	1	2048	223291	223661	223661	31751	223661	31751	223201	32673
300	25	3	2	2058	209940	210981	210981	37309	210981	37309	210294	38448
300	25	3	3	2090	209621	210910	210910	37753	210910	37753	210910	37753
300	25	3	4	2104	214773	215639	215732	36872	215732	36872	215273	37754
300	25	3	5	2045	211567	212432	212432	31295	212432	31295	212432	31295
300	25	5	1	1229	162952	163668	163746	15309	163746	15309	163746	15309
300	25	5	2	1234	151533	152860	152951	15539	152951	15539	152699	16109
300	25	5	3	1254	152043	153347	153489	14987	153489	14987	153017	15725
300	25	5	4	1262	155179	156340	156340	17136	156430	17136	156172	17754
300	25	5	5	1227	153592	154936	154936	14621	154936	14621	154324	15734
300	25	10	1	614	107525	109400	109400	4846	109400	4864	109314	5181
300	25	10	2	617	100699	102306	102383	4303	102400	4303	**102424***	**4303***
300	25	10	3	627	102338	103707	103794	4415	103832	4872	103832	4872
300	25	10	4	631	103177	105290	105294	4294	105294	4294	105294	4294
300	25	10	5	613	102649	104120	104218	5137	104218	5137	104125	5190
300	75	10	2	567	266877	267728	268003	6185	268007	5999	**268102***	**6185***

From Table 1, one can observe that the proposed method is able to reach new dominated points and matches the rest of the bounds achieved by more recent method in the literature.

4 Conclusion

In this study, the bi-objective quadratic multiple knapsack problem was solved by using an ϵ-constraint strategy-based method. Such a method combines both local branching and ϵ-constraint strategies. The local branching tries to intensify the search process while the ϵ-constraint strategy is used for diversifying the search process. The preliminary computational results showed that the method remains competitive when comparing its provided results to those achieved by the best methods available in the literature.

Références

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